Rational Inattention, Menu Costs, and Multi-Product Firms: Micro Evidence and Aggregate Implications*

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Abstract

Using a New Zealand firm-level survey, I show that firms producing more goods have both better information about inflation and more frequent but smaller price changes. To explain these empirical findings, I develop a general equilibrium menu cost model with rationally inattentive multi-product firms. I show that the interaction of nominal and informational rigidities leads to a new selection effect: price adjusters are better informed than non-adjusters. This selection endogenously generates a leptokurtic distribution of desired price changes, which amplifies monetary non-neutrality. Compared to a one-product baseline, the real effects of monetary shocks are 12% smaller in a two-product model.

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1 Introduction

The way firms set their prices and form their expectations has important implications for the transmission of monetary policy shocks. The extensive empirical literature studying detailed microdata on firms’ beliefs and pricing behavior has found that many firms are not fully aware of macroeconomic conditions and change their prices infrequently.\(^1\) Economic models have embedded realistic features of price setting and firms’ expectations formation, such as costly price adjustments and a lack of awareness of economic conditions, to study the ability of monetary policy to stimulate the economy. Most of those models, however, assume firms produce only one product.\(^2\) This assumption can be justified if multi-product firms act approximately like a collection of single-product firms, a view that has been difficult to challenge due to a dearth of empirical evidence regarding multi-product firms’ economic decisions. A second justification is that there are significant computational challenges in integrating multi-product pricing into menu cost and rational inattention models. This paper challenges both justifications by first providing new empirical evidence that multi-product firms price their products and form their expectations differently and then developing a new theoretical model in which multi-product pricing under both nominal and informational rigidities has important implications for monetary policy transmission.

This paper builds on a growing literature studying how firms form their expectations and process information. For example, Kumar et al. (2015), Coibion et al. (2018a), and Afrouzi (2020) study various factors that determine how much attention firms devote to tracking macroeconomic conditions. My new empirical contribution to this literature is to document that the number of products that firms produce is an important determinant of firm-level inattention to macroeconomic conditions: firms with a greater product scope have better information about aggregate economic conditions. Moreover, I investigate how firms’ product scopes affect their price-setting decisions and show that firms with a greater product scope have more frequent but smaller price changes. The joint characterization of the relationship between firms’ product scopes and their decisions regarding both price setting and information acquisition complements previous literature studying microdata on multi-product pricing (e.g. Lach and Tsiddon, 1996; Bhattacharai and Schoenle, 2014; Stella, 2018; Bonomo et al., 2019a).

This paper also contributes to our understanding of monetary non-neutrality by using a new general equilibrium model with both nominal and informational rigidities in a world of multi-

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\(^1\)See, for instance, Klenow and Malin (2010) and Nakamura and Steinsson (2008, 2013) for comprehensive reviews about micro price stickiness. See Kumar et al. (2015), Coibion et al. (2018a), Boneva et al. (2019), and Afrouzi (2020) for evidence on pervasive inattention on the firm side to macroeconomic variables.

product pricing. This model builds on two strands of monetary models with multi-product firms to capture micro-evidence on economies of scope. First, menu cost models with multi-product firms, such as Midrigan (2011) and Alvarez and Lippi (2014), exhibit economies of scope in price setting: firms with a greater number of products have more frequent but smaller price changes. Second, rational inattention models with multi-product firms, such as Pasten and Schoenle (2016), exhibit economies of scope in information processing: firms with a greater number of products have a large incentive to acquire and process information about aggregate shocks. My theoretical contribution is to combine all three elements—menu costs, rational inattention, and multi-product firms—in a unified framework to study the ability of monetary policy to stimulate the economy. This model, disciplined by micro-evidence, serves to quantitatively study how monetary non-neutrality is affected 1) by firms’ product scopes and 2) by the interaction between menu costs and rational inattention regardless of firms’ product scopes.

My starting point is to explore the empirical characteristics of multi-product firms in terms of their price-setting and information acquisition decisions. To do so, I use a representative survey of New Zealand firms’ macroeconomic beliefs. My empirical analysis documents that firms’ product scopes are systematically related to their inattention to macroeconomic conditions and price-setting decisions. First, I find that firms with a greater number of goods are better informed about aggregate inflation. Firms make systematically smaller errors about recent values of aggregate inflation when they produce a greater number of products. Moreover, firms with a greater product scope are also willing to pay more for information about future inflation. This finding implies that firms with more products have incentives to process more information about macroeconomic conditions. To the best of my knowledge, these results are the first empirical evidence documenting differential information acquisition decisions of firms based on the number of products they sell. Second, I show that firms with a greater number of products have more frequent but smaller price changes. This finding is consistent with the previous empirical literature that used different micro price data. Jointly, these results illustrate that the scope of products sold by firms affects both their information acquisition and price-setting decisions.

What are the aggregate implications of the micro-evidence that I show above for the ability of monetary policy to stimulate the economy? To answer this question, I develop a new model that captures the behavior of firms in the survey. Specifically, I assume that it is costly for firms to observe and process information regarding underlying shocks. Firms optimally choose their information set given the costs of information. This captures the pervasive inattention among

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3 Appendix Figure G.1 shows how my model fits within the literature on menu costs and rational inattention models.
firms in the survey. Second, firms have to pay a fixed menu cost to reset their prices, which leads to the infrequent price changes observed in the survey data. I further assume that this fixed cost is independent of how many prices firms change. This assumption introduces economies of scope in price setting: firms with greater product scope change their prices more frequently and by smaller amounts because the average costs of changing prices are lower for them. Finally, I assume that firms face two types of shocks—idiosyncratic good-specific shocks and aggregate monetary shocks—and the marginal cost of processing information is independent of the firms’ number of products. This assumption introduces economies of scope in information processing: firms with a greater product scope want to learn more about aggregate monetary shocks because information about them can be used to price all their goods.5

I embed this setup of firm decision making into a full-fledged dynamic general equilibrium model and study the macroeconomic implications for monetary non-neutrality. I use three target moments from the survey data to discipline the model parameters: the frequency and size of price changes and the slope of the backcast error curve on the number of products.6 The first two help calibrate the menu cost parameter and the size of idiosyncratic shocks, while the third helps calibrate the informational cost parameter. I focus on two key questions with this general equilibrium model. First, I explore how the interaction between rational inattention and menu costs affects firms’ optimal decisions and, therefore, how the economy responds to monetary shocks, regardless of firms’ product scopes. Second, I show how firms’ product scopes affect monetary non-neutrality through economies of scope in multi-product firms by comparing the macroeconomic dynamics to monetary shocks in the one-good vs. two-good versions of my model.

My first theoretical finding is that the baseline one-good version of the model generates large real effects of monetary policy shocks that are seven times larger than those in the standard menu cost model and nearly as large as those in the Calvo sticky price model. Standard menu cost models have small and short-lived real effects of monetary shocks due to the strong selection effects of price changes: an expansionary monetary shock triggers numerous price increases that originate from far below the average level and offset many price decreases.7 The extent of the selection effects depends on the underlying distribution of firms’ desired price changes. When the

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5This model nests the baseline menu cost and rational inattention models as special cases. For example, without informational cost, this model coincides with the menu-cost-only models with either single-product firms (e.g., Golosov and Lucas, 2007) or multi-product firms (e.g., Midrigan, 2011; Alvarez and Lippi, 2014). Without menu cost, this model nests the rational-inattention-only models with either single-product firms (e.g., Maćkowiak and Wiederholt, 2009) or multi-product firms (e.g., Pasten and Schoenle, 2016).

6The slope of the backcast error curve captures the magnitude of the decrease in firm-level errors regarding recent values of the growth rate of nominal GDP if the number of products that firms produce increases by one unit.

7Gagnon et al. (2013) study the effect of large inflationary shocks on the timing of price changes using Mexican CPI data and find direct support for a selection effect. Carvalho and Kryvtsov (2018) find evidence of strong price selection across goods and services using detailed micro-level consumer price data for the UK, the US, and Canada.
distribution is Gaussian, many prices are clustered around the adjustment margins, leading to large price selection effects. Previous menu cost models that try to explain the strong non-neutrality of money found in the data have often assumed that the distribution of idiosyncratic shocks has excess kurtosis and a fat tail, so the majority of desired price changes are near zero while some of them are very far from zero (e.g., Gertler and Leahy, 2008; Midrigan, 2011; Vavra, 2013; Karadi and Reiff, 2019; Baley and Blanco, 2019). My new contribution to this literature is to show that the interaction between menu costs and rational inattention frictions can endogenously generate a distribution of firms’ desired price changes with excess kurtosis. I show that this leptokurtic distribution of desired price changes weakens the selection effects of price changes, amplifying the impact response of output to monetary shocks by 23% in my baseline one-good version of the model compared with standard menu cost models with fully informed firms.

How does the interaction between nominal and informational rigidities generate the leptokurtic distribution of desired price changes? As highlighted in previous models with both information and nominal rigidities, when both acquiring information and adjusting prices are costly, firms’ optimal price-setting rules depend on their own subjective uncertainty (e.g. Gorodnichenko, 2008; Afrouzi and Yang, 2021b). Firms with large uncertainty regarding underlying shocks have a wider inaction region for pricing decisions, as they want to wait and see until they get more information to resolve their uncertainty. This leads to another kind of selection effect regarding information processing: price adjusters have better information than non-adjusters. A new finding of this paper is that this selection effect in information processing endogenously leads to a leptokurtic distribution of firms’ desired price changes, with the majority near zero and some far away from zero.

The second theoretical finding is that the real effects of monetary policy shocks decrease by 12% in the two-good version of the model compared with the one-good version of the baseline model. As I discussed above, the two-good version of the model exhibits two types of economies of scope in multi-product firms. First, there are economies of scope in price setting. Because paying the menu cost allows firms to change the prices of all of their goods simultaneously, there are many small and large price changes in the two-good version of the model. This weakens selection effects of price changes and should tend to amplify the real effects of monetary policy shocks in the two-good version of the model. Second, I also find significant economies of scope in information processing: firms in the two-good version of the model have better information and

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8The real option value and the "wait-and-see" rule have been studied in the literature on the effects of steady-state uncertainty (e.g., Dixit and Pindyck, 1994; Abel and Eberly, 1999) or second-moment uncertainty shocks (e.g., Bloom, 2009; Vavra, 2013; Gilchrist et al., 2014; Bloom et al., 2018). Unlike this literature, the interaction between information and nominal rigidities makes firms’ optimal pricing rules depend on their own subjective uncertainty.

9In Appendix F, I provide evidence on the leptokurtic distribution of desired price changes using the New Zealand survey data. Moreover, I show that firms with greater subjective uncertainty expect a longer duration before their next price changes. This finding is direct evidence of the "wait-and-see" rule in firms’ price-setting decisions.
lower uncertainty about monetary policy shocks than firms in the one-good version of the model. Because multi-product firms learn about monetary policy shocks rapidly, this force will tend to reduce the real effects of nominal shocks. The quantitative analysis shows that cumulative output effects are smaller in the two-good version of the model than the one-good version of the model. This implies that the scope motive in information processing quantitatively dominates its effect on pricing decisions, thereby leading to reduced effects of nominal shocks on economic activity as we move to a multi-product environment.

My results are robust to introducing higher numbers of products in the model. I simplify the baseline model by assuming that firms’ information acquisition decisions are independent of their price-setting decisions. Although this assumption eliminates the interesting interaction between nominal and informational rigidities, the simplified version of the model shares the core predictions of the baseline model: firms with a greater product scope have better information about aggregate shocks, and the kurtosis of the distribution of price changes increases with the number of products that firms produce. Consistent with the results from the full baseline model, I find that the cumulative output responses in the simplified version of the model decrease with the number of products sold by firms. This result implies that in the class of rational inattention models, the ratio of kurtosis and the frequency of price changes might not be a sufficient statistic, which is derived by Alvarez et al. (2016), for the real effects of monetary shocks.

The paper is organized as follows. Section 2 empirically evaluates how firms’ attentiveness to aggregate inflation and price-setting behavior are related to their number of products. In Section 3, I develop a new menu cost model with rationally inattentive multi-product firms that captures the behavior of firms in the survey, and I study firms’ optimal decisions regarding information acquisition and price setting. In Section 4, I extend my model to show how product scope affects firm behavior and monetary policy transmission. Section 5 concludes the paper.

2 Empirical Evidence

In this section, I empirically explore how firms’ product scopes relate to 1) their attentiveness to aggregate economic conditions and 2) the frequency and size of their price changes. To this end, I use a quantitative survey of firms’ expectations toward macroeconomic conditions in New Zealand. See Coibion et al. (2018a) and Kumar et al. (2015) for a comprehensive description of the survey.
to previous studies that used the same survey.\textsuperscript{11} First, firms producing a greater number of products are better informed about current aggregate inflation. Second, both the duration and average size of their price changes decrease with the number of products firms produce.

\section*{2.1 Firms’ Product Scopes in the Survey Data}

I use the second wave of the survey, which was implemented between February and April 2014, to identify the number of products firms produce. In the survey, firms’ managers were asked about the number of products they sell in addition to their main product or product line. Appendix Table G.1 shows the summary statistics regarding firms’ number of products by industry. The median is 9, but it is 7 when firms in retail and wholesale trade sectors are excluded. In the baseline regressions, I exclude these retail and wholesale trade firms because their strategies for pricing and information processing are likely to be different from those of firms in other sectors, such as manufacturing and service industries.\textsuperscript{12} In the data, a large fraction of firms (about 18\% of all firms) compared with other studies sell only one product or have one product line.\textsuperscript{13} There are two reasons behind the large fraction of single-product firms in this survey. First, the firms included in the survey were relatively small, with the average number of employees being about 31 and the largest number being about 600 employees. Second, the survey question is about the number of products or product lines at a firm. As there might be several similar types of products in a product line, this question captures firms’ perceptions of the unit of their product scope. In fact, I find that the average of firms’ output shares of their main product (or product line) is about 60\% excluding single-product firms, implying that firms define their unit of product scope a bit broadly.

\section*{2.2 Number of Products and Attentiveness to Inflation}

I first investigate the relationship between the firms’ number of products and their attentiveness to current aggregate economic conditions. Firms’ attentiveness to aggregate economic conditions, measured by their backcast error in aggregate inflation, is defined to capture firms’ knowledge about the current aggregate economy. Given that recent aggregate economic conditions are largely

\textsuperscript{11}Several papers use the data to characterize how firms form their expectations. For example, Afrouzi (2020) shows that strategic complementarity decreases with competition, and documents that firms with more competitors have more certain posteriors about the aggregate inflation. Also, Coibion et al. (2018b) evaluate the relation between first-order and higher-order expectations of firms, including how they adjust their beliefs in response to a variety of information treatments.

\textsuperscript{12}Including retail and wholesale trade firms in the sample does not change the baseline results that I show later. See, for example, Appendix Table G.3.

\textsuperscript{13}For example, Bhattarai and Schoenle (2014) document that 98\% of all prices are set by firms with more than one good in the microdata that underlie the calculation of the U.S. PPI.
Figure 1: Number of Products and Attentiveness to Aggregate Inflation

Notes: The left panel plots percentile of firms’ number of products versus the average of firm backcast errors about past 12 month inflation within each percentile. The right panel plots the percentile of firms’ number of products versus the average of willingness to pay for a professional inflation forecast. The willingness to pay is measured from answers to the following question in Wave #4 survey: “How much would you pay per year to have access to a monthly magazine of professional forecasts of future inflation?” Black lines are linear fitted lines and shaded areas are 90% confidence intervals. The size of bins represents average size of employment of firms in each percentile.

observable in real time, I define the backcast error as the absolute values of the difference between the actual past 12 months of aggregate inflation and managers’ corresponding beliefs from the survey. As documented in Coibion et al. (2018a), firms are not well informed about current aggregate inflation, resulting in 4.5% backcast errors on average.

In addition, I find that firms’ backcast errors are related to their number of products. Figure 1 shows there is a clear positive relationship between the number of products firms produce and their attentiveness to aggregate inflation. The left panel shows that firms with a smaller product scope produce larger backcast errors on average. In the right panel, I show the relationship between firms’ product scopes and their willingness to pay for professional forecasts about future inflation from the fourth wave of the survey. The latter is another measure of firms’ incentives to be attentive to the aggregate economy. Here, I find a positive correlation between the number of products firms produce and their willingness to pay for professional inflation forecasts, which implies that firms

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14 The Consumer Price Index is used to calculate the actual past 12 month aggregate inflation. The baseline results are quantitatively similar when I use the GDP deflator or the Producer Price Index to calculate the actual inflation rate.

15 One might have a concern that firms do not know what the inflation means. However, Kumar et al. (2015) document that 86% of firm managers in the survey could correctly explain what inflation means and they believed that statistical agencies were credible in measuring price changes. Coibion et al. (2018a) also highlight that the large errors are not driven by specific language about the definition of inflation used in the survey.

16 See Appendix Table G.2 for the summary statistics of the firms’ backcast error is aggregate inflation by the quartiles of firms’ product scopes within different industries.
Table 1: Number of Products and Knowledge about Aggregate Inflation

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<tr>
<td>Panel A. Dependent variable: Absolute value of actual minus firm-reported inflation in prior 12 months</td>
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<tr>
<td>log(number of products)</td>
<td>-0.314**</td>
<td>-0.207***</td>
<td>-0.580***</td>
<td>-0.252***</td>
</tr>
<tr>
<td>(0.148)</td>
<td>(0.059)</td>
<td>(0.150)</td>
<td>(0.060)</td>
<td></td>
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<tr>
<td>Observations</td>
<td>593</td>
<td>582</td>
<td>448</td>
<td>440</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.341</td>
<td>0.801</td>
<td>0.344</td>
<td>0.900</td>
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<tr>
<td>Panel B. Dependent variable: Willingness to pay for professional inflation forecasts</td>
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<tr>
<td>log(number of products)</td>
<td>5.717**</td>
<td>3.805***</td>
<td>6.965**</td>
<td>3.960**</td>
</tr>
<tr>
<td>(2.565)</td>
<td>(1.186)</td>
<td>(2.635)</td>
<td>(1.643)</td>
<td></td>
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<tr>
<td>Observations</td>
<td>381</td>
<td>368</td>
<td>328</td>
<td>320</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.242</td>
<td>0.657</td>
<td>0.289</td>
<td>0.697</td>
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<tr>
<td>Firm-level controls</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Industry fixed effects</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
<td></td>
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<tr>
<td>Manager controls</td>
<td></td>
<td></td>
<td>Yes</td>
<td>Yes</td>
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Notes: This table reports results for the Huber robust regression. Dependent variables are the absolute value of firm errors about past 12 month inflation from Wave #1 survey (Panel A) and firms’ willingness to pay for professional forecasts about future inflation from Wave #4 (Panel B). Firm-level controls include log of firms’ age, log of firms’ employment, foreign trade share, number of competitors, firms’ beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm’s manager), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit Australian and New Zealand Standard Industrial Classification (ANZ SIC) level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

with larger product scopes are likely to pay more attention to aggregate conditions.

One potential concern is that this negative correlation between the number of products firms produce and their knowledge of current aggregate inflation is driven by other firm-level characteristics. For example, as big firms are likely to have a larger product scope and a larger capacity to process information, it might seem that the negative correlation stems from the size of the firm rather than its product scope.\(^7\) To address this concern, I regress firms’ inattention to inflation, as measured by 1) their absolute errors in regard to recent inflation rates and 2) their willingness to pay for professional future inflation forecasts. I use a log of the firms’ number of products, controlling for firm-level characteristics such as a log of firm age, a log of total employment, foreign trade share, the firms’ number of competitors, their beliefs about price differences relative to their competitors, and the slope of the profit function.\(^8\) Column (1) of Table 1 shows that firms with a

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\(^7\)Kaihatsu and Shiraki (2016) show that the size of firms significantly affects differences in their inflation expectations and Frache and Lluberas (2019) find that large firms have lower forecast errors in inflation than small firms.

\(^8\)The slope of a firm’s profit function is calculated as the ratio of the extent to which a firm could increase its profit (as a percent of revenue) if it could reset its price freely at the time of the survey to the percent price change the firm would implement if it could reset its price freely at the time of the survey.
large number of products are likely to make small errors about current aggregate inflation and are more willing to pay more for professional forecasts about future aggregate inflation. Column (2) shows the significant negative correlation after controlling for industry fixed effects.

Another potential concern is that the survey respondents—here the managers of firms—have different abilities or incentives to pay attention to aggregate economic conditions. To address this issue, in Column (3), I report regression results after controlling for managers’ characteristics, such as age, education, income level, and number of years at their firms. Again, after controlling for manager characteristics, I find a negative correlation between the firms’ number of products and their knowledge of or attentiveness to aggregate inflation.

2.3 Number of Products and Size and Frequency of Price Changes

In this subsection, I document the relationship between the number of products and the frequency and size of their price changes. Firms’ managers were asked about the duration and size of their expected next price changes. After controlling for firms’ characteristics and their incentives for changing their prices, this question quantifies the frequency and size of firms’ price changes.

Table 2 shows the relationship between the firms’ number of products and the frequency and size of their price changes. Panel A shows that, after controlling for firm-level characteristics, the duration of price changes is negatively correlated with the number of products. This negative correlation is even stronger when I control for managers’ characteristics. In Panel B, I also present that after controlling for industry fixed effects, there is a negative correlation between the number of products firms produce and the size of their price changes. This result shows that, conditional on the price change, firms with a greater product scope change their prices by smaller amounts.

2.4 Summary and Relation to Monetary Models

In this section, I show that firms with a greater number of products have both better information about aggregate inflation and more frequent but smaller price changes. Can existing monetary

19For example, Tanaka et al. (2019) show that managers’ GDP forecasting ability is linked to their management ability and experience.

20In Appendix Table G.4, I show that firms’ backcast errors in the growth rate of nominal GDP also decrease with their number of products. In the general equilibrium model I study in Section 4, I calibrate the information cost parameter to match the slope coefficient of the regression of the backcast errors in the growth rate of nominal GDP.

21Previous studies have also found negative correlations between the firms’ number of products and the duration and size of their price changes. For example, using U.S. PPI microdata, Bhattacharai and Schoenle (2014) show that firms with larger product scopes are more likely to change their prices frequently, and, conditional on price changes, they change by smaller amounts. Parker (2017) also identified this negative correlation between firms’ product scopes and the duration and size of their price changes using other survey data on New Zealand firms.
Table 2: Number of Products and Duration and Size of Price Changes

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<tr>
<td><strong>Panel A. Dependent variable: Duration of expected next price changes</strong></td>
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<tr>
<td>log(number of products)</td>
<td>-0.169</td>
<td>-0.205*</td>
<td>-0.276*</td>
<td>-0.474***</td>
</tr>
<tr>
<td></td>
<td>(0.144)</td>
<td>(0.116)</td>
<td>(0.142)</td>
<td>(0.139)</td>
</tr>
<tr>
<td>Observations</td>
<td>588</td>
<td>580</td>
<td>440</td>
<td>445</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.495</td>
<td>0.604</td>
<td>0.520</td>
<td>0.548</td>
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</table>

| **Panel B. Dependent variable: Size of expected next price changes** |         |         |         |         |
| log(number of products) | -0.111  | -0.318*** | -0.113  | -0.239*** |
|                      | (0.066) | (0.089) | (0.079) | (0.081) |
| Observations         | 578     | 578     | 432     | 431     |
| R-squared            | 0.073   | 0.612   | 0.075   | 0.423   |

Firm-level controls: Yes, Yes, Yes, Yes
Industry fixed effects: Yes, Yes
Manager controls: Yes, Yes

Notes: This table reports results for the Huber robust regression. Dependent variables are the duration of expected next price changes from Wave #1 (Panel A) and the (absolute) size of expected next price changes from Wave #1 survey (Panel B). Firm-level controls include log of firms’ age, log of firms’ employment, foreign trade share, number of competitors, firms’ beliefs about price difference from competitors, and the slope of the profit function. Industry fixed effects include dummies for 14 sub-industries excluding retail and wholesale trade sectors. Manager controls include the age of the respondent (each firm’s manager), education, income, and tenure at the firm. Sample weights are applied to all specifications. Robust standard errors (clustered at the 3-digit ANZ SIC level) are reported in parentheses. ***, **, * denotes statistical significance at 1%, 5%, and 10% levels respectively.

models with multi-product firms explain both empirical findings? In fact, rational inattention models with multi-product firms, such as Pasten and Schoenle (2016), predict that a negative correlation between the firms’ number of products and their inattentiveness to aggregate inflation. In the presence of both good-specific shocks and aggregate shocks, multi-product firms want to be more informed about the aggregate shocks, as they will affect the marginal costs of all their products. On the other hand, menu cost models with multi-product firms, such as Midrigan (2011) and Alvarez and Lippi (2014), predict that multi-product firms change their prices more frequently and by less because menu cost technology can introduce economies of scope in price setting.

However, those two models are not capable of explaining both findings simultaneously by assumption. Rational inattention models assume flexible prices to focus on the effects of information rigidity while menu cost models assume perfect information to focus on the effects of nominal rigidity. Those two assumptions are inconsistent with the empirical evidence that firms change their prices infrequently and are not fully aware of macroeconomic conditions.

Moreover, the previous literature finds a contradictory implication of firms’ product scopes for monetary non-neutrality. In menu cost models, the output effects of monetary shocks increase with
the firms’ number of products, as selection effects of price changes from menu cost technology
decrease with the firms’ number of products.\textsuperscript{22} In contrast, in rational inattention models, the real
effects of monetary shocks \textit{decrease} with the number of products firms produce because firms with
a greater product scope are more informed about monetary policy shocks.\textsuperscript{23}

In sum, neither model with multi-product firms can account for the empirical relationship be-
tween firms’ product scopes and their decisions regarding both price setting and information ac-
quision, and they have contradictory implications regarding firms’ product scopes for monetary
non-neutrality. This calls for a new model that is disciplined by the empirical findings from micro-
data in order to study the macroeconomic implications of monetary non-neutrality.

3 Price Setting with Menu Costs for a Rationally Inattentive Multi-Product Firm

In this section, I develop a menu cost model for a rationally inattentive multi-product firm. Be-
fore constructing a full-fledged dynamic general equilibrium model in the next section, I consider
the decision problem of this firm that faces both good-specific shocks and an aggregate monetary
shock. The firm chooses a set of optimal signals about the underlying shocks given costs of infor-
mation and pays a single fixed cost to reset all of its prices. The goal of this section is to explore
how the firm’s decisions regarding information acquisition and price setting are affected by 1) the
interaction between menu costs and rational inattention and 2) the firm’s product scope .

3.1 A Rationally Inattentive Firm’s Problem

Consider a multi-product firm that produces \( N \) goods, indexed by \( j = 1, 2, \cdots, N \). The firm sets
its price of good \( j \), \( p_{j,t} \), to match a (frictionless) optimal price, \( p_{j,t}^* \). Suppose its optimal price
of good \( j \) consists of two components, a good-specific shock, \( a_{j,t} \), and an aggregate shock, \( m_t \),
\( p_{j,t}^* = a_{j,t} + m_t \). I assume that both shocks follow random walk processes:

\[
a_{j,t} = a_{j,t-1} + \varepsilon_{j,t}^a, \quad \varepsilon_{j,t}^a \sim N(0, \sigma_a^2) \text{ for } j = 1, 2, \cdots, N,
\]

\[
m_t = m_{t-1} + \varepsilon_t^m, \quad \varepsilon_t^m \sim N(0, \sigma_m^2),
\]

\textsuperscript{22}Alvarez et al. (2016) show that in menu cost models, the cumulative output response to a monetary shock increases
in the number of products, \( N \), and is given by \( \frac{3N}{N+2} \).

\textsuperscript{23}In Appendix B.1, I show that in a rational inattention model, the cumulative response of output to a monetary
shock is only a function of firms’ average subjective uncertainty about the monetary shock and decreases with the
firms’ number of products.
where $\varepsilon_{j,t}^a$ and $\varepsilon_{m,t}^m$ are independent and identically distributed.\textsuperscript{24} A flow loss of the firm in profits is the sum of the distance between its price of each good and the frictionless price, $B \sum_{j=1}^{N} (p_{j,t} - p_{j,t}^*)^2$, where $B$ captures the concavity of the profit function with respect to each price.\textsuperscript{25}

This firm is rationally inattentive. At the beginning of each period, the firm has to choose how precisely it wants to observe its current set of (frictionless) optimal prices subject to a cost of information processing. Formally, at time $t$, the firm chooses a set of signals about both good-specific and aggregate shocks from a set of available signals, $S_t = \{S_{j,t}^a\}_{j=1}^{N} \cup S_{t}^m$, such that

$$S_{j,t}^a = \{a_{j,t} + \eta_{j,t} \xi_{j,t}^a : \eta_{j,t} \geq 0, \xi_{j,t}^a \sim N(0,1)\}, \quad \text{for} \quad j = 1, 2, \cdots, N,$$

$$S_{t}^m = \{m_{t} + \eta_{m,t} \xi_{t}^m : \eta_{m,t} \geq 0, \xi_{t}^m \sim N(0,1)\},$$

where $\{\xi_{j,t}^a\}_{j=1}^{N}$ and $\xi_{t}^m$ are the firm’s rational inattention errors. Let $S_{t-1}$ be the firm’s information set at the beginning of period $t$ before it receives new signals about its frictionless optimal prices. At each time $t$, given $S_{t-1}$, the firm chooses a set of its signals $s_{j,t}^a \in S_{j,t}^a$ for $j = 1, 2, \cdots, N$, and $s_{t}^m \in S_{t}^m$ subject to the cost of information processing. Then, the firm’s information set evolves as $S_t = S_{t-1} \cup s_t$ where $s_t = \{s_{j,t}^a\}_{j=1}^{N}, s_{t}^m\}$. The evolution of information set implies that the firm does not forget information over time. This "no-forgetting constraint" implies that the current information choice has a continuation value and thus the optimal information choice is a solution of a dynamic information acquisition problem.

I assume that the cost of information is linear in Shannon’s mutual information function. The firm pays $\psi$ units of its (per-good) revenue for every bit of expected reduction in uncertainty, where uncertainty is measured by entropy. Denote this cost as $\psi I(s_t; \{p_{j,t}^*\}_{j=1}^{N}|S_{t-1})$, which will be defined later. At each period, based on its optimal information choice, the firm chooses whether to change its prices. I assume that the firm can change all prices by paying a single fixed cost, $\theta$.\textsuperscript{26}

Figure 2 shows the timing of events for the firm’s problem: at the beginning of period $t$, the firm starts with an a priori information set, $S_{t-1}$, and forms a prior over its optimal prices at that time. Then it chooses a new set of signals, $s_t$, subject to the cost of information processing and updates its information set, $S_t$. Given this time $t$ information set, the firm decides whether to change its prices and pay the fixed cost $\theta$ or to wait until the next period without changing its prices. If the firm decides to change its prices, it also chooses how much it changes the prices. Thus, the firm optimally chooses a set of signals about the underlying shocks and prices ($p_{j,t}$) over

\textsuperscript{24}The random walk process of the underlying shock is a common assumption in the menu cost literature, as it simplifies the firm’s problem by making it choose its price gaps, which are defined by the difference between the frictionless and the actual prices. See, among others, Barro (1972), Tsiddon (1993), and Alvarez and Lippi (2014).

\textsuperscript{25}While I take this characteristic as an assumption, this loss function can be derived as a second order approximation to a twice-differentiable profit function around the non-stochastic steady state.

\textsuperscript{26}In Appendix E, I discuss some evidence of the firm-specific menu costs using the survey data. I also discuss the implications of adding product-specific menu costs in the model.
Choose a set of optimal signals, $s_t$, given a cost function $\psi I(s_t; \{p_{j,t}^*\}_{j=1}^N|S^{t-1})$.

Choose a price of good $j$, $p_{j,t}: \{(p_{k,t}^*)_{k=1}^N, S^t\} \rightarrow \mathbb{R}$, given menu costs, $\theta$.

Figure 2: Timing of Events for a Firm’s Problem

**Notes:** This figure shows a sequence of events in each period of the model.

time, contingent on the evolution of its beliefs.

Formally, the firm’s problem is as follows:

$$
\min_{\{p_{j,t}\}_{j=1}^N, s_t} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t \left( B \sum_{j=1}^N (p_{j,t} - p_{j,t}^*)^2 + \theta 1_{\{\text{for any } j, p_{j,t} \neq p_{j,t-1}\}} \right) \right]
$$

loss from suboptimal prices

$$
+ \psi \mathcal{I}(s_t; \{p_{j,t}^*\}_{j=1}^N|S^{t-1}) \right] S^{-1} \right]
$$

cost of information processing

subject to $p_{j,t}^* = a_{j,t} + m_t, \quad \forall j = 1, 2, \cdots, N$,

$S^t = S^{t-1} \cup s_t$, $S^{-1}$ is given,

where $1_{\{\text{for any } j, p_{j,t} \neq p_{j,t-1}\}}$ is an indicator function that is 1 if it changes any one of its prices.\(^{27}\)

**Cost of Information Processing** The cost of information processing is linear in Shannon’s mutual information function. Let $\mathcal{H}(X|Y)$ be a conditional entropy of a random variable of $X$ given knowledge of $Y$. The firm’s flow cost of information at time $t$ is $\psi \mathcal{I}(s_t; \{p_{j,t}^*\}_{j=1}^N|S^{t-1})$, where $\mathcal{I}(s_t; \{p_{j,t}^*\}_{j=1}^N|S^{t-1}) = \mathcal{H}(\{p_{j,t}^*\}_{j=1}^N|S^{t-1}) - \mathbb{E}[\mathcal{H}(\{p_{j,t}^*\}_{j=1}^N|S^{t-1})|S^{t-1}]$ is the reduction in uncertainty about its (frictionless) optimal prices that the firm experiences by observing the set of signals, $s_t$, given its prior information set, $S^{t-1}$, and $\psi$ is the marginal cost of a bit of information.

\(^{27}\)Besides the existence of menu costs, this problem is different from the previous rational inattention models in LQG settings, such as Maćkowiak and Wiederholt (2015) or Pasten and Schoenle (2016), which solve the problem by assuming that the cost of information is not discounted and optimizing at the long-run steady state for the information structure. In contrast, I assume that the firm discounts future costs of information at the same discount rate as its payoffs and solve the dynamic rational inattention problem. This dynamic nature of firms’ information acquisition is the reason that firms are subject to the no-forgetting constraints. See, for instance, Afrouzi and Yang (2021a) for a detailed discussion of solutions for the dynamic rational inattention problem in LQG setups.
Let \( z_{j,t}^a \equiv \text{var}(a_{j,t}|S^t) \) and \( z_{t}^m \equiv \text{var}(m_t|S^t) \) be the firm’s subjective uncertainty about the \( j \)-good-specific shock and about the aggregate shock, respectively. I can rewrite the cost of information processing at time \( t \) in terms of \( \{z_{j,t}^a\}_{j=1}^N \) and \( z_{t}^m \):

\[
\mathcal{I}(s_t; \{p_{j,t}^*,\}_{j=1}^N|S^{t-1}) = \sum_{j=1}^N \mathcal{I}(s_{j,t}^a; a_{j,t}|S^{t-1}) + \mathcal{I}(s_t^m; m_t|S^{t-1})
\]

\[
= \frac{1}{2} \sum_{j=1}^N \log_2 \left( \frac{z_{j,t}^a + \sigma_a^2}{z_{j,t}^a} \right) + \log_2 \left( \frac{z_{m,t} + \sigma_m^2}{z_{m,t}} \right)
\]  

(2)

where \( \{z_{j,-1}^a\}_{j=1}^N \) and \( z_{-1}^m \) are given. The first equality follows from the fact that the underlying shocks are independent and the firm observes independent signals about them. The second equality holds because the firm observes Gaussian signals about the underlying shocks, which are also Gaussian. Moreover, I can rewrite the no-forgetting constraint, \( S^t = S^{t-1} \cup s_t \), in terms of the firm’s subjective uncertainty, \( 0 \leq z_{j,t}^a \leq z_{j,t-1}^a + \sigma_a^2 \) for \( j = 1, 2, \cdots, N \) and \( 0 \leq z_{t}^m \leq z_{t-1}^m + \sigma_m^2 \).

This reformulation shows that the cost of information processing is directly related to how much each firm reduces its subjective uncertainty about the good-specific shocks and the aggregate shock, given their prior uncertainty about those shocks. If the marginal cost of information processing, \( \psi \), is zero, the firm would like to choose zero subjective uncertainty and thus there is no contemporaneous loss from imperfect information. In this case, the firm’s problem is identical to the problem in a standard menu cost model with multi-product firms.

On the other hand, if there is no informational cost, \( \psi = 0 \), then the firm chooses zero subjective uncertainty about the \( j \)-good-specific shock and about the aggregate shock, respectively.

Recursive Formulation of the Firm’s Problem  I reformulate the firm’s problem (1) in a recursive form to characterize its optimal decision rules and simulate the model numerically. Let \( x_{j,t} = p_{j,t} - \mathbb{E}[p_{j,t}^*|S^t] \) be the firm’s perceived price gap of product \( j \). Then, the loss from suboptimal prices can be decomposed into two components:

\[
\mathbb{E} \left[ (p_{j,t} - p_{j,t}^*)^2 \middle| S^t \right] = \underbrace{\frac{z_{j,t}^a + z_{t}^m}{z_{j,t}^a}}_{\text{contemporaneous loss from imperfect information}} + \underbrace{x_{j,t}^2}_{\text{contemporaneous loss from nominal rigidities}},
\]  

(3)

where \( z_{j,t}^a \) and \( z_{t}^m \) are the subjective uncertainty about the \( j \)-good-specific shock and about the aggregate shock, respectively.

On the one hand, if there is no informational cost, \( \psi = 0 \), then the firm chooses zero subjective uncertainty about the \( j \)-good-specific shock and thus there is no contemporaneous loss from imperfect information. In this case, the firm’s problem is identical to the problem in a standard menu cost model with multi-product firms. On the other hand, if there is no menu cost, \( \theta = 0 \), then the firm can always adjust its prices freely.
and thus will choose zero perceived price gaps, $x_{j,t} = 0$ for all $j$. In this case, the firm’s problem is identical to the problem in a standard rational inattention model with multi-product firms.

Notice that the perceived price gaps, $\{x_{j,t}\}$, are the firm’s choice variables when it decides to change its prices. However, if the firm does not want to change its prices, then the perceived price gaps are stochastic variables that evolve according to $x_t \sim N(x_{t-1}, \Sigma_t)$ where $x_t = \{x_{1,t}, x_{2,t}, \cdots, x_{N,t}\}'$ and

$$
\Sigma_t(j, k) = \begin{cases} 
  z^m_{t-1} + \sigma^2_m - z^m_l & \text{if } j \neq k \\
  z^a_{j,t-1} + \sigma^2_a - z^a_{j,t} + z^m_{t-1} + \sigma^2_m - z^m_l & \text{if } j = k.
\end{cases} \tag{4}
$$

Given (2), (3), and (4), I reformulate the firm’s problem (1) in a recursive form with $2N + 1$ state variables and occasionally binding constraints:

$$
V(\{x_{j,-1}\}_{j=1}^N, \{z^a_{j,-1}\}_{j=1}^N, z^m_1) = \max_{\{z^a_j\}_{j=1}^N, z^m} \mathbb{E} \left[ \max \left\{ V^I(\{x_{j}\}_{j=1}^N, \{z^a_{j}\}_{j=1}^N, z^m), V^C(\{z^a_{j}\}_{j=1}^N, z^m) \right\} \right] \\
- \frac{\psi}{2} \left( \sum_{j=1}^N \log_2 \left( \frac{z^a_{j-1} + \sigma^2_a}{z^a_j} \right) + \log_2 \left( \frac{z^m_{t-1} + \sigma^2_m}{z^m_t} \right) \right) \mathbb{I}^{-1},
\begin{array}{l}
0 \leq z^a_j \leq z^a_{j-1} + \sigma^2_a, \ \forall j = 1, 2, \cdots, N, \\
0 \leq z^m \leq z^m_{t-1} + \sigma^2_m,
\end{array}
$$

$$
V^I(\{x_{j}\}_{j=1}^N, \{z^a_{j}\}_{j=1}^N, z^m) = -B \sum_{j=1}^N (x^2_j + z^2_a_j + z^m) + \beta V(\{x_{j}\}_{j=1}^N, \{z^a_{j}\}_{j=1}^N, z^m)
\begin{array}{l}
\text{with } x \sim N(\mu, \Sigma), \ and
\end{array}
$$

$$
V^C(\{z^a_{j}\}_{j=1}^N, z^m) = \max_{\{y_j\}_{j=1}^N} -B \sum_{j=1}^N (y^2_j + z^2_a_j + z^m) - \theta + \beta V(\{y_{j}\}_{j=1}^N, \{z^a_{j}\}_{j=1}^N, z^m).
$$

Here $V^I(\{x_{j}\}_{j=1}^N, \{z^a_{j}\}_{j=1}^N, z^m)$ represents the firm’s value of not changing its prices. Similarly, $V^C(\{z^a_{j}\}_{j=1}^N, z^m)$ is the firm’s value of changing its prices.

### 3.2 Decision Rules

In this section, I describe key properties of the firm’s optimal decision rules. First, because of the quadratic objective function and the symmetry of the normal distribution, the value function is also symmetric around the null vector for the perceived price gaps. Second, given the optimal choices of subjective uncertainty about the good-specific shocks and about the aggregate shock, the value function is decreasing in the absolute values of perceived price gaps. These two properties imply
that, given optimal choices of subjective uncertainty, the firm chooses to have zero perceived price gaps for all their goods whenever it decides to change its prices by paying the menu cost, $\theta$.

Because the firm’s problem is a non-convex optimization problem in its price-setting decision and there are occasionally binding no-forgetting constraints for its choices of subjective uncertainty, it needs to be solved numerically. Using the method of value function iteration, I solve the problems of two types of firms: a single-product firm and a two-product firm. I first investigate how the interaction between menu costs and rational inattention frictions affects the single-product firm by characterizing its optimal information acquisition and price-setting decisions. Then, I show how economies of scope in both price setting and information processing affect the two-product firm by comparing with the single-product firm.

### 3.2.1 A Single-Product Firm

I first consider a single-product firm’s optimal decision rules. Here I drop the $j$-index because the firm produces only one product.

**Optimal Information Acquisition** Optimal policy rules for choosing a firm’s subjective uncertainty about the good-specific shock are presented in Figure 3. In particular, the left panel of Figure 3 shows that when its prior uncertainty is low enough and its prior price gap is close to zero, the no-forgetting constraint binds and the firm does not acquire new information. The amount of information acquisition increases in both the firm’s prior uncertainty ($z_{t-1}$) and the distance between its current price and the frictionless optimal price ($|x_{t-1}|$). In other words, the firm has a large incentive to collect and process information when the firm is quite uncertain about the realization of the underlying shocks and thinks that it is likely that it will need to change prices. This is because potential losses from mistakes in the price-setting decisions are large if the firm thinks that it is likely that it will need to change its price. The firm could make wrong decisions either by paying the menu cost and changing the price when it was not supposed to do so or by choosing not to change its price when it should.

---

28 The most computationally burdensome part is to compute firms’ expected future values. When the firm does not change prices, its price gaps are stochastic variables that are jointly normally distributed with a mean vector, $x_{t-1}$, which is the firm’s state variable, and a covariance matrix, $\Sigma$, which is its choice variable. Standard approximation methods for the transition probability of states, such as Tauchen (1986), are not applicable, as approximation errors are quite large. I compute the expected value of the firms’ value functions using an explicit numerical integration technique. See Appendix C for the detailed description.

29 To illustrate this example, I set $\theta = 0.0074$, $\psi = 0.0035$, $\sigma_a = 0.0168$, and $\sigma_m = 0.0044$. These are the parameters that I calibrate when I solve a general equilibrium single-product model in Section 4.
Figure 3: Information Acquisition and Inaction Bands of a Single-Product Firm

Notes: The left panel plots the amounts of information acquisition from a single-product firm’s optimal choice of subjective uncertainty about the good-specific shock. The right panel shows inaction bands of a single-product firm as a function of its subjective uncertainty. Different lines represent the inaction bands with different levels of subjective uncertainty about the aggregate shock ($z^m_t$). Black lines are the inaction bands of a myopic firm whose discount factor is zero. Since this myopic firm does not care about a continuation value of information, the subjective uncertainty is not its state variable, which leads the inaction bands of this firm to be constant.

**Price-Setting Decision Given the Optimal Information Choices** After choosing optimal signals about the underlying shocks and forming the new information set, the firm decides whether to change its price, based on the new information set. Due to the fixed menu cost of adjusting prices, the firm adopts $S-s$ rules in setting its prices, like in standard menu cost models. There are adjustment thresholds ($s, S$) such that if the firm’s perceived price gap is greater than $S$ or less than $s$, it pays the fixed cost and adjusts decreases its price to the frictionless optimal level. The adjustment thresholds are the firm’s inaction bands. One interesting feature in this model is that the inaction bands are time-varying. Formally, let $\hat{x}_t$ be the firm’s posterior belief about its perceived price gap after observing the new optimal signals and before changing its price at time $t$. Then,

$$\hat{x}_t = p_{t-1} - \mathbb{E}[a_t + m_t|S^t] = x_{t-1} - \{K^a_t (s^a_t - \mathbb{E}[a_t|S^{t-1}]) + K^m_t (s^m_t - \mathbb{E}[m_t|S^{t-1}])\},$$

where $K^a_t$ and $K^m_t$ are the optimal Kalman gains for the good-specific shock and for the aggregate shock, respectively. A higher value of the Kalman gains implies that the firm chooses to observe more precise signals about the underlying shocks. Given the firm’s optimal choice of subjective uncertainty, $z^a_t$ and $z^m_t$, there exists an adjustment threshold $\tilde{x}(z^a_t, z^m_t) \geq 0$ such that

$$-\tilde{x}(z^a_t, z^m_t)^2 + \beta V(B\tilde{x}(z^a_t, z^m_t), z^a_t, z^m_t) = -\theta + \beta V(0, z^a_t, z^m_t).$$

18
The firm will change its price if \(|\hat{x}_t| > \bar{x}_t(z^a_t, z^m_t)\). Then, the perceived price gap at the end of period \(t\), \(x_t\), is

\[
x_t = \begin{cases} 
\hat{x}_t & \text{if } |\hat{x}_t| \leq \bar{x}_t(z^a_t, z^m_t) \\
0 & \text{if } |\hat{x}_t| > \bar{x}_t(z^a_t, z^m_t).
\end{cases}
\]

The right panel of Figure 3 shows the inaction bands, \((-\bar{x}_t(z^m_t), \bar{x}_t(z^m_t))\), for the various values of \(z^m_t\). With \(\beta > 0\), the inaction bands vary with the firm’s subjective uncertainty.\(^{30}\) When the firm is more uncertain about the underlying shocks, the inaction bands are wider. This makes sense because when the firm is uncertain about the underlying shocks, it is optimal to wait until it gets more information about them.

The main implication of the interaction between menu costs and rational inattention frictions is that the firm is likely to be more informed about the underlying shocks when it changes its price than when it does not. As will be clear in the next section, in a general equilibrium model with a large number of firms, this interaction leads to a selection effect of information processing such that price adjusters are more informed about the underlying shocks than non-adjusters.\(^{31}\)

### 3.2.2 A Two-Product Firm

Now, I consider the two-product firm’s optimal information acquisition and price setting decision. In fact, the two-product firm shares the same characteristics about its optimal decision rules with the single-product firm that I discussed earlier. However, two interesting motives based on economies of scope emerge in the two-product firm’s optimal choices—economies of scope in price changes through the menu cost technology and economies of scope in information processing through the rational inattention friction.

**Economies of Scope in Price Changes** Figure 4 shows a two-product firm’s information acquisition and price setting behavior in the model simulation. Like a single-product firm, the inaction bands of the two-product firm also depend on its subjective uncertainty. The main difference in

\(^{30}\)The inaction bands in a myopic model (\(\beta = 0\)) are constant and given by \((-\sqrt{\theta/B}, \sqrt{\theta/B})\) because the firm’s subjective uncertainty is no longer a state variable for the firm’s problem.

\(^{31}\)Models with both menu costs and observational costs such as Alvarez et al. (2011) and Bonomo et al. (2019b) also have similar implications. In these models, a firm has to pay a fixed cost to acquire full-information about the underlying shocks and can change its price by paying a fixed menu cost. The optimal pricing rule implies that the firm only changes its price when it pays the observational cost, which is an extreme case of the selection effect of information processing in the sense that the firm has full information when it changes its price while it does not acquire new information at all otherwise. Gorodnichenko (2008) also shows that firms have an incentive to buy an additional signal prior to changing prices in a model with menu costs and endogenous information choice.
Figure 4: A Two-Product Firm in Model Simulation

Notes: The upper left panel plots a two-product firm’s subjective uncertainty about both a good-specific shock (red dash-dot line) and an aggregate shock (blue solid line). The upper right panel plots the firm’s perceived price gap ($x_t$) for good 1, its true price gap for good 1 under perfect information, and inaction bands for good 1. The lower right panel plots the firm’s perceived price gap ($x_t$) for good 2, its true price gap for good 2 under perfect information, and inaction bands for good 2. The firm changes its price when the perceived price gaps are out of the inaction bands. The lower left panel plots these price changes for both goods.

the two-product firm’s price setting decision is that the price change of one of its products depends on the perceived price gap of the other product. For example, the right upper and lower panels of Figure 4 show that when the perceived price gap of the second product is large, the inaction bands for the first product are narrow. This relationship implies that the timing of price changes within the firm is synchronized and, more importantly there are both large and small price changes. This is called economies of scope in price changes from the menu cost technology. If the firm decides to pay the menu cost to change one of its prices, then the price change of the other products is free for the firm. As a result, given the single menu cost, the two-product firm is likely to change its prices more frequently than the single-product firm. Moreover, as the additional price change is free and thus there are many small price changes, the two-product firm changes its prices, on average, by a smaller amount than the single-product firm. These results are consistent with my empirical finding that firms producing more goods have more frequent but smaller price changes. As highlighted in the menu cost literature, economies of scope in price changes will weaken the selection effects of price changes by letting some price changes be random (e.g. Midrigan 2011,
Alvarez and Lippi 2014). The weak selection effects of price changes in an economy with a large number of multi-product firms will then lead to an amplified real effect of monetary shocks.

**Economies of Scope in Information Processing** The two-product firm is also different from the single-product firm in terms of its optimal information acquisition about the aggregate shock. In particular, I find that given the same marginal cost of information processing, the two-product firm is more informed about the aggregate shock than the single-product firm. The average subjective uncertainty about the aggregate shock for the two-product firm is 25% smaller than that for the single-product firm. Because the firm’s optimal prices for all goods are affected by the aggregate shock, the value of information about the aggregate shock will be higher if the firm produces more products.\(^{32}\) This result is consistent with my empirical finding that firms that produce more goods have better information about aggregate inflation. As the two-product firm has lower subjective uncertainty about the aggregate shock than the single-product firm, the two-product firm responds more strongly to monetary policy shocks by learning about them more rapidly and therefore changing their prices more rapidly. In the economy with a large number of firms, this scope motive in information processing acts as a strong force to weaken monetary non-neutrality.

In sum, the two-product firm’s information acquisition and price-setting decisions show two economies of scope motives that work in opposite directions for monetary non-neutrality. The question is how quantitatively large each scope motive is. To draw the implications of firms’ product scopes for monetary non-neutrality in the model with both menu costs and rational inattention friction, we need to extend the model in a general equilibrium setup and discipline it with micro-evidence. This is the goal of the next section.

### 4 A Dynamic General Equilibrium Model

In this section, I extend the menu cost model with rationally inattentive multi-product firms of Section 3 to a dynamic general equilibrium model. The model is disciplined using micro-evidence of Section 2 and then used for quantitative analysis on the transmission of monetary shocks. The focus of this analysis is twofold. First, I investigate how the interaction between rational inattention

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\(^{32}\)This relationship is clearly seen if we consider the firm’s problem without menu costs (\(\theta = 0\)). In this case, the firm’s optimal subjective uncertainty about the aggregate shock satisfies the following FOC:

\[
B \cdot N = \frac{\psi}{2 \log 2} \left( \frac{1}{z^m_t} - \beta \frac{1}{z^m_t + \sigma^2_m} \right),
\]

where \(z^m_t\) is decreasing in the number of products, \(N\). See Appendix B for the optimal solutions for the rationally inattentive firm’s problem without menu costs.
and menu costs affects the distribution of firms’ desired price changes as well as the distribution of subjective uncertainty. I show these two distributions are important determinants of monetary non-neutrality. Second, I compare the two-good and one-good versions of the model to study how multi-product pricing affects the real effects of monetary shocks through economies of scope motives in both information processing and price setting that I studied in the previous section.

4.1 Environment

The economy is populated by a representative household and a unit measure of monopolistically competitive firms. Each firm sells \( N \) products. I first discuss the household problem and then present the firm problem and define equilibrium.

**Households** The representative household consumes a Dixit-Stiglitz aggregate consumption, \( C_t \), of a basket of multiple goods \( j \in \{1, 2, \ldots, N\} \) purchased from firms \( i \in [0, 1] \), and supplies labor \( L_t \) to maximize the expected lifetime utility with a discount factor \( \beta \in (0, 1) \).

The representative household’s problem is

\[
\max_{\{C_{i,j,t}\}_{j=1}^N, C_t, L_t, B_t} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t U(C_t, L_t) \right],
\]

subject to

\[
\int \left( \sum_{j=1}^N P_{i,j,t} C_{i,j,t} \right) dt + B_t \leq R_{t-1} B_{t-1} + W_t L_t + \Pi_t, \quad \text{for all } t,
\]

where \( C_t = \left( \frac{1}{N} \sum_{j=1}^N C_{j,t}^{\gamma+1} \right)^{\frac{1}{\gamma+1}} \) and \( C_{j,t} = \left( \int (A_{i,j,t} C_{i,j,t}) \frac{1}{\varepsilon+1} dt \right)^{\frac{\varepsilon+1}{\varepsilon}} \). Here \( \mathbb{E}_t [\cdot] \) is the full-information rational expectation operator at time \( t \). As the main purpose of this paper is to study the effects of nominal rigidity and rational inattention among firms, I assume that the household is fully informed about all prices and wages. \( B_t \) is the demand for nominal bonds and \( R_{t-1} \) is the nominal interest rate. \( L_t \) is the labor supply of the household, \( W_t \) is the nominal wage, and \( \Pi_t \) is the aggregate profit from the firms. \( C_t \) is the aggregator over the consumption for differentiated goods and \( C_{j,t} \) is an aggregator over the consumption of good \( j \). \( A_{i,j,t} \) is the quality of the good \( j \) produced by firm \( i \). Higher \( A_{i,j,t} \) increases the marginal utility of consumption for that good while it also increases the production cost for that good, as I describe below.\(^{33}\) \( \varepsilon \) is the constant elasticity of substitution across different firms that produce the same good and \( \gamma \) is the constant elasticity of substitution across different goods.

\(^{33}\)The assumption that idiosyncratic shocks affect both the cost at which a good is sold and the household’s marginal utility for the good is common in the menu cost literature to reduce the dimensionality of the state space and thus the computational burden (see, e.g., Midrigan, 2011; Alvarez and Lippi, 2014; Mongey, 2017; Karadi and Reiff, 2019).
The above formulation implies that demand for good \( j \) produced by firm \( i \) is
\[
C_{i,j,t} = A_{i,j,t}^{\varepsilon^{-1}} \left( \frac{P_{i,j,t}}{P_{j,t}} \right)^{-\varepsilon} \left( \frac{P_{j,t}}{P_t} \right)^{-\gamma} C_t,
\]
where \( P_t \) is the price of the aggregate consumption \( C_t \), and \( P_{j,t} \) is the price index for good \( j \). These prices are given by
\[
P_t = \left( \frac{1}{N} \sum_{j=1}^{N} P_{j,t}^{1-\gamma} \right)^{\frac{1}{1-\gamma}} \quad \text{and} \quad P_{j,t} = \left( \int_0^1 \left( \frac{P_{i,j,t}}{A_{i,j,t}} \right)^{1-\varepsilon} \, dt \right)^{\frac{1}{1-\varepsilon}}.
\]

**Firms** There is a measure one of firms, indexed by \( i \), that operate in monopolistically competitive markets. Each firm produces \( N \) goods, indexed by \( j \). Firms take wages and demands for their goods as given, and choose their prices \( \{P_{i,j,t}\}_{j=1}^N \) based on their information set, \( S_{t,i} \), at that time. After setting their prices, firms hire labor from a competitive labor market and produce the realized level of demand that their prices induce with a production function for good \( j \),
\[
Y_{i,j,t} = A_{i,j,t} \left( \frac{P_{i,j,t}}{W_t} \right)^{\varepsilon^{-1}} \left( \frac{P_{j,t}}{P_t} \right)^{-\gamma} C_t,
\]
where \( Y_t \) is the nominal aggregate demand.

At each period, firms optimally decide their prices and signals subject to the costs of changing prices and of processing information. First, changing the price entails a fixed cost, \( \tilde{\theta} \). I express this cost as a fraction \( \theta \) of the steady-state frictionless revenue from selling one of \( N \) products. This cost is incurred when at least one price is changed and is independent of the total number of prices that the firm adjusts. Second, firms choose their optimal information set by taking into account the cost of obtaining and processing information. At the beginning of period \( t \), firm \( i \) wakes up with its initial information set, \( S_{t,i}^{-1} \). Then it chooses optimal signals, \( s_{i,t} \), from a set of available signals, \( S_{i,t} \), subject to the cost of information, which is linear in Shannon’s mutual information function. Denote \( \tilde{\psi} \) as the marginal cost of information processing. Again, I express this marginal cost as a fraction \( \psi \) of the steady-state frictionless revenue from selling one of \( N \) products. Firm \( i \) forms a new information set, \( S_i^t = S_{i}^{t-1} \cup s_{i,t} \), and uses it to set its new prices, \( \{P_{i,j,t}\}_{j=1}^N \).

Firm \( i \) chooses a set of signals to observe over time \( (s_{i,t} \in S_{i,t})_{t=0}^\infty \) and a pricing strategy that maps the set of its prices at \( t - 1 \) and its information set at \( t \) to its optimal price at any given period,
\( P_{i,j,t} : \{ P_{i,j,t-1}\}_{j=1}^N, S_t^i \} \rightarrow \mathbb{R} \) where \( S_t^i = S_{t-1}^i \cup s_{i,t} = S_{t-1}^i \cup \{ s_{i,t}\}_{t=0}^t \) is the firm’s information set at time \( t \). Then, firm \( i \)'s problem is to maximize the net present value of its lifetime profits given an initial information set:

\[
\max_{\{ s_{i,t} \in S_{i,t} \}, \{ P_{i,j,t} \}_{j=1}^N, \{ W_t \}_{t=0}^\infty} \mathbb{E} \left[ \sum_{t=0}^\infty \beta^t \Lambda_t \left( \Pi_{i,t}(\{ P_{i,j,t}, A_{i,j,t}, P_{j,t}\}_{j=1}^N, W_t, P_t, Y_t) \right) \right] \left| S_t^{-1} \right]
\]

\[-\tilde{\theta} \mathbf{1}_{\{ \text{for any } j, P_{i,j,t} \neq P_{i,j,t-1} \}} - \tilde{\psi} \mathcal{I}(s_{i,t}, \{ A_{i,j,t}\}_{j=1}^N, W_t | S_t^{-1}) \]

subject to

\( S_t^i = S_{t-1}^i \cup s_{i,t} \),

where \( \Lambda_t = \frac{U_{c,t}}{U_{c,0}/P_0} \) and \( \mathcal{I}(s_{i,t}, \{ A_{i,j,t}\}_{j=1}^N, W_t | S_t^{-1}) \) is Shannon’s mutual information function.\(^{34}\)

**Monetary Policy and Equilibrium** Money supply is equal to nominal spending:

\[
\int \left( \sum_{j=1}^N P_{i,j,t} C_{i,j,t} \right) dt = P_tC_t = M_t,
\]

and the log of money supply, \( m_t \equiv \log(M_t) \), follows a random walk process:

\[
m_t = m_{t-1} + \varepsilon_t^m, \quad \varepsilon_t^m \sim N(0, \sigma_m^2),
\]

where \( \varepsilon_t^m \) is an independently and identically distributed normal disturbance.

An equilibrium consists of an allocation for the representative household, \( \Omega^H \equiv \{ C_t, \{ C_{i,j,t}\}_{j=1}^N, L_t, B_t \}_{t=0}^\infty \), an allocation for every firm \( i \in [0, 1] \), \( \Omega^F_i \equiv \{ s_{i,t} \in S_{i,t}, \{ P_{i,j,t}, L_{i,j,t}, Y_{i,j,t}\}_{j=1}^N \}_{t=0}^\infty \), a set of prices \( \{ \{ P_{j,t}\}_{j=1}^N, P_t, R_t, W_t \}_{t=0}^\infty \), and a stationary distribution over firms’ states such that (i) all firms optimize, (ii) the household optimizes, (iii) the stationary distribution is consistent with actions, and (iv) all markets clear. Appendix A.3 precisely defines an equilibrium of this model.

### 4.2 Computing the Equilibrium

I assume the representative household’s preferences of the form \( U(C, L) = \log(C) - L \). The log-utility of consumption implies that the intertemporal optimal condition relates nominal interest rate to the law of motion of aggregate demand. This assumption enables me to formulate monetary policy in terms of either nominal interest rates or aggregate nominal demand. Moreover, linear

\(^{34}\)While I implicitly assume that firms’ signals are informative only about the idiosyncratic good-specific shocks and aggregate nominal wage, this signal structure is optimal after taking the second-order approximation to the firms’ profit function as I do when solving the problem. Up to the second-order approximation, only the aggregate nominal wage and the idiosyncratic shocks matter for the firm’s profit function. Thus, this assumption is without loss of generality.
disutility in labor ensures that nominal wage is proportional to nominal aggregate demand, \( W_t = P^*_t C_t = M_t \). This closely follows Golosov and Lucas (2007) and Midrigan (2011) and makes monetary shocks translate one-for-one into changes in the firms’ nominal marginal costs.

Firms’ profit functions (6) imply that without any frictions in price setting and in information processing, firm \( i \)’s frictionless optimal price of good \( j \), \( P^*_i,j,t \), is a constant markup over its nominal marginal cost, \( P^*_i,j,t = \frac{\varepsilon}{\varepsilon-1} W_t A_{i,j,t} \). Let \( \mu_{i,j,t} = \frac{P_i,j,t}{P^*_i,j,t} \) be firm \( i \)’s true price gap for good \( j \), which is a gap between the actual price and the frictionless optimal price. Here \( \bar{\mu} = \frac{\varepsilon}{\varepsilon-1} \) is a non-stochastic steady-state level of the price gap. Then, firm \( i \)’s nominal flow profit at time \( t \) can be written as a function of the set of these price gaps, \( \sum_{j=1}^{N} (\mu_{i,j,t} - 1) (\mu_{i,j,t})^{-\varepsilon} (W_t)^{1-\varepsilon} (P_{j,t})^{\varepsilon-\gamma} (P_t)^{\gamma} Y_t \).

To solve the firm’s problem, I take the second-order approximation to the firms’ profit function and derive firms’ losses from sub-optimal pricing. I assume that the set of available signals, \( S_{i,t} \), has the following properties. First, the firm chooses \( N + 1 \) independent signals for each shock, implying that paying attention to aggregate conditions and paying attention to good-specific idiosyncratic conditions are separate activities. Second, each signal is Gaussian. Third, all noise in signals is idiosyncratic and independent. The second-order approximation reduces the state space of the problem from an entire distribution to its covariance matrix. Moreover, because the signals are Gaussian and the objective function is quadratic, it enables me to focus on a Gaussian posterior. Under these assumptions, each firm’s problem is identical to that studied in Section 3.1.

I compare two economies with \( N = 1 \) and \( N = 2 \). I use the method of value function iteration to solve the firms’ problem and simulate the economy with a large number of firms for a long period to make sure that the economy reaches a stationary distribution over firms’ states.

4.3 Calibration and Parameterization

In the numerical exercises, I set the monthly discount factor to \( \beta = 0.96^{(1/12)} \), which implies a real interest rate of 4 percent. I set the elasticity of substitution across firms to be four (\( \varepsilon = 4 \)), which matches the firms’ average markup of 33% in the survey data. Moreover, I assume the elasticity

35See Appendix A for the detailed derivation of the second-order approximation and the recursive representation for the firms’ problem.
36See Appendix E for a discussion on the implications and limitations of these assumptions.
37Because the number of state variables is increasing linearly with the number of products as \( 2N + 1 \), a model with more than two products is hard to solve. A two-product economy is also considered as the baseline in Midrigan (2011) and Karadi and Reiff (2019). Moreover, in the New Zealand survey data, the average of the main product’s share of total output value is about 60%, excluding single-product firms. It implies that a two-product firm might be a good benchmark. In Section 4.7, I solve models with any arbitrary number of products under some simplifying assumptions.
38This value is in the middle of 3 and 7, the elasticity of substitution parameters in Midrigan (2011) and Golosov and Lucas (2007), respectively. It directly affects the slopes of profit curves, and thus the estimates of menu costs and the standard deviation of good-specific shocks, without altering the main findings.
Table 3: Data and Model Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Single-product model</th>
<th>Two-product model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median (absolute) size of price changes</td>
<td>0.0576</td>
<td>0.0576</td>
<td>0.0576</td>
</tr>
<tr>
<td>Median frequency of price changes</td>
<td>0.0833</td>
<td>0.0833</td>
<td>0.0833</td>
</tr>
<tr>
<td>Slope of the backcast error curve</td>
<td>-0.020</td>
<td></td>
<td>-0.020</td>
</tr>
</tbody>
</table>

Notes: The table presents moments of the data and simulated series from the single- and two-product models parameterized at the baseline values in Table 4. To get the slope of the backcast error curve, I regress the absolute value of firm errors about past 12 month nominal GDP growth rate from Wave #4 survey on the number of products each firm produces. Regression results are reported in Appendix Table G.4. See Section 4.3 for details.

of substitution between goods is the same as that across firms ($\gamma = 4$). However, the value of $\gamma$ plays little role, as there are no common good-specific shocks in the model.

I calibrate the standard deviation of the log difference in nominal demand, $\sigma_m$, to match the standard deviation of the growth rate of nominal GDP in New Zealand, 0.0044.\(^{39}\) There are three key model parameters that should be calibrated: the size of menu costs ($\theta$), the size of marginal costs of information processing ($\psi$), and the size of idiosyncratic good-specific shocks ($\sigma_a$). I assume that the marginal cost of processing one bit of information in both one-good and two-good versions of the model are the same.\(^{40}\) I calibrate these three parameters to match the median frequency of price changes (once a year), the median size of absolute price changes (5.76%), and slope of backcast errors in the growth rate of aggregate nominal GDP on the number of products (-0.02) observed in the survey data.\(^{41}\)

The latter is obtained by regressing the firms’ backcast errors in the growth rate of nominal GDP on the firms’ number of products.\(^{42}\) The estimate shows that, controlling for firm and manager characteristics as well as industry controls, the backcast errors decrease by 0.02 percentage points when firms produce one more good.\(^{43}\) The model counterpart measure is calculated by taking the difference between average backcast errors in the growth rate of nominal demand in single- and two-product models. The three moments exactly identify the three key model parameters, and, as Table 3 shows, all the targeted moments are well matched.

Table 4 shows the calibrated and assigned parameters in both single-product and two-product

---

39I restrict the sample to post-1991 data, as New Zealand has explicitly targeted inflation in that time period.

40As shown in Pasten and Schoenle (2016), an alternative assumption for the cost of information processing, such as a constant loss per good from imperfect information, does not change the main findings in this paper.

41Parker (2017) also finds that the median frequency of price changes is once a year in New Zealand using 2010 Business Operations Survey data carried out by Statistics New Zealand, though the data does not provide a quantitative measure of the size of price changes.

42The backcast errors in the nominal GDP growth rate are from the fourth wave of the survey. In the survey, firms’ managers are asked about current inflation and the real GDP growth rate in New Zealand. I take a summation of both measures to obtain firms’ perceived growth rate of nominal GDP in the economy.

43See Appendix Table G.4 in for the regressions results with and without controls.
Table 4: Calibration and Assigned Parameters

<table>
<thead>
<tr>
<th>Panel A. Calibrated parameters</th>
<th>Single-product model</th>
<th>Two-product model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Menu cost ($\theta$)</td>
<td>0.0074</td>
<td>0.0281</td>
</tr>
<tr>
<td>Information cost ($\psi$)</td>
<td>0.0035</td>
<td>0.0035</td>
</tr>
<tr>
<td>Standard deviation of idiosyncratic shocks ($\sigma_a$)</td>
<td>0.0183</td>
<td>0.0188</td>
</tr>
<tr>
<td>Standard deviation of monetary policy shocks ($\sigma_m$)</td>
<td>0.0044</td>
<td>0.0044</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Assigned parameters</th>
<th>Single-product model</th>
<th>Two-product model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount factor ($\beta$)</td>
<td>0.9966</td>
<td>0.9966</td>
</tr>
<tr>
<td>Elasticity of substitution across firms ($\varepsilon$)</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Elasticity of substitution between goods ($\gamma$)</td>
<td>4.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

Notes: The table presents the baseline parameters for the general equilibrium models with single- and two-product firms. Panel A shows the calibrated parameters which match the three key moments shown in Table 3. Panel B shows the assigned parameters. See Section 4.3 for details.

models. The baseline parameterization implies a menu cost of 0.74 percent of steady-state (per good) revenue in the single-product model. Given the average frequency of price changes, the overall cost of price adjustment in the single-product model is around 0.062 percent of steady-state revenue. Similarly, the overall cost of price adjustment in the two-product model is around 0.058 percent of steady-state revenue. These values are smaller than estimates in the previous literature, which often used U.S. data, as the average size of absolute price changes in New Zealand is small.\(^{44}\)

The calibrated standard deviations of the good-specific shocks are around 1.7 percent per month in both models, which are about four times bigger than the standard deviation of the monetary shock. The calibrated cost of information processing, $\psi/\mathcal{I}(\cdot; \cdot)$, is 0.11 percent of steady-state (per good) revenue. The cost is relatively smaller than menu costs and this small cost implies that imperfect information models do not require large information costs to match the data. The implied average Kalman gains on the signal about idiosyncratic shocks are 0.27 in both models (see Table 5). This is equivalent to a quarterly gain of 0.60, which is slightly higher than the estimate of 0.50 in Coibion and Gorodnichenko (2015), which uses the U.S. Survey of Professional Forecasters data.

4.4 Simulation

In this section, I show the simulation results for two models with a large number of firms.\(^{45}\) I emphasize two distributional characteristics that will be important to understand the sizes of the

\(^{44}\)For example, Levy et al. (1997) find menu costs of 0.7 percent of revenue, while Zbaracki et al. (2004) find price adjustment costs as large as 1.2 percent. Stella (2018) estimates the total cost of changing prices to be between 0.3% and 1.3% of revenues.

\(^{45}\)A simulation algorithm for the two-product model is presented in Appendix C.1.
Table 5: Average Kalman Gains in Models

<table>
<thead>
<tr>
<th></th>
<th>Single-product model</th>
<th>Two-product model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Signal about</td>
<td>Signal about</td>
</tr>
<tr>
<td>Average Kalman gains</td>
<td>good-specific shocks</td>
<td>monetary shocks</td>
</tr>
<tr>
<td></td>
<td>($K_{gt}^{p}$)</td>
<td>($K_{mt}^{pp}$)</td>
</tr>
<tr>
<td>All firms</td>
<td>0.267</td>
<td>0.089</td>
</tr>
<tr>
<td>- Price adjusters</td>
<td>0.611</td>
<td>0.257</td>
</tr>
<tr>
<td>- Price non-adjusters</td>
<td>0.235</td>
<td>0.074</td>
</tr>
<tr>
<td></td>
<td>0.279</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>0.653</td>
<td>0.259</td>
</tr>
<tr>
<td></td>
<td>0.244</td>
<td>0.120</td>
</tr>
</tbody>
</table>

Notes: The table presents average Kalman gains across firms in both baseline models. Column (1) and (3) show the average Kalman gains on the signal about the good-specific shocks in the single-product model and those in the two-product model, respectively. Column (2) and (4) show the average Kalman gains on the signal about the monetary policy shock in the single-product model and those in the two-product model, respectively.

real effects of the monetary shocks in both models. First, I show that selection in information processing endogenizes a leptokurtic distribution of desired price changes, which acts as a force to weaken selection effects of price changes. Second, multi-product firms value more information about the monetary policy shock than the single-product firms.

Selection in Information Processing and Endogenous Leptokurtic Distribution of Price Gaps

Table 5 shows an important characteristic about firms’ optimal information choices. The second and third rows compare the average Kalman gain of firms that adjust their prices to those of firms that do not adjust their prices. The Kalman gain represents how much weight firms put on new information relative to their prior estimates. Thus, the average Kalman gains can be interpreted as the average degree of firms’ attentiveness to the underlying shocks. I find that there is a selection in information processing: price adjusters are more informed about both idiosyncratic and aggregate shocks than price non-adjusters. In particular, the price adjusters’ average Kalman gains in the signals about idiosyncratic shocks are about three times bigger than those of price non-adjusters. These findings are true in both the single- and the two-product models, implying that selection in information processing operates regardless of the number of products in the model.

This selection in information processing is due to the interaction between firms’ optimal information and pricing decisions. As shown in Section 3.1, firms’ optimal information acquisition policies are affected by their beliefs about their price gaps. If a firm believes that its price is far away from its optimal level, the potential losses from mistakes in pricing decisions would be very large, which makes the firm process more information about the shocks to reduce losses. After the realization of shocks, this firm is more likely to be a price adjuster because its prior price gap

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46 When firms’ signals are perfectly telling about the true underlying shocks, the Kalman gain is 1. When firms optimally choose not to be perfectly informed due to the cost of information, the Kalman gain is less than 1.
Figure 5: Distributions of True and Perceived Price Gaps in the Single-Product Model

Notes: This figure plots distributions of price gaps in the baseline single-product model. Black lines are the average of inaction bands across firms in the model. At the beginning of period, before the realization of shocks, all firms believe that their price is within the inaction bands. Blue bar graph shows the distribution of firms’ prior about their price gaps \( p_{i,j,t-1} - E_{t-1}[p_{i,j,t}^* \mid S_{t-1}] \) at the beginning of period. After the Gaussian shocks realized, firms’ marginal costs change, and thus their true price gap \( p_{i,j,t-1} - p_{i,j,t}^* \) also changes. Black dashed line shows the distribution of these true price gaps. Firms choose their optimal signals about the shocks and form a new posterior about their (frictionless) optimal price. Then, the posterior of perceived price gap is \( p_{i,j,t} - E_t[p_{i,j,t}^* \mid S_t] \). Red dash-dot line shows the distribution of these perceived price gaps.

is close to the inaction bands. For this reason, on average, the price adjusters in the economy are better informed about the underlying shocks than the price non-adjusters.

This new selection mechanism in information processing endogenously generates a leptokurtic distribution of firms’ perceived desired price changes.\(^{47}\) Figure 5 shows the distributions of the perceived and true price gaps in the single-product model. First, the blue line is a prior distribution about firms’ perceived price gaps. At the beginning of the period, all firms believe that their prices are within their inaction bands and there are many zero perceived price gaps of zero, which were adjusted at the previous period. This implies that the prior distribution of firms’ perceived price gaps is very concentrated around zero and has a high kurtosis. After being hit by Gaussian idiosyncratic shocks, the distribution of true price gaps is Gaussian (red dashed line). If firms have perfect information about their true optimal prices, their pricing decisions would be based on their true price gaps. As the distribution of the true price gaps is Gaussian, as in the standard single-

\(^{47}\)If a firm has a price gap of \( x \% \) and it is free to change its price, then it would change by \( -x \% \). I use "price gaps" and "desired price changes" interchangeably.
product menu cost model, there would be large selection effects of price changes: an expansionary monetary shock triggers many large price increases, but it offsets a mass of large price decreases.

However, in the model with both menu costs and informational costs, firms are rationally inattentive about their true optimal prices. Firms all choose their optimal Gaussian signals and update their estimates of price gaps, but they do not do so in the same way. Firms that think that their price gap is well within their inaction bands and that think it is unlikely that they will need to change prices have little incentive to collect much new information: they choose to remain quite uninformed and update the estimates of their price gaps with a large weight on their (imprecise) priors. In contrast, firms that think they are close to the boundaries of their inaction regions have a high incentive to collect information and therefore choose to become more informed. Because the distribution of priors of firms’ perceived price gaps is very concentrated around zero, this selection in information processing makes the distribution of posteriors of the perceived gaps leptokurtic.

This leptokurtic distribution implies a small selection effect of price changes because the rationally inattentive firms’ pricing decisions are based on their posterior of perceived price gaps. Thus, the endogenous leptokurtic distribution will act as a strong force to amplify monetary non-neutrality in the general equilibrium model. Previous studies of menu cost models often assume exogenously a leptokurtic distribution of shocks (e.g., Gertler and Leahy, 2008; Midrigan, 2011; Vavra, 2013; Karadi and Reiff, 2019; Baley and Blanco, 2019). Unlike these studies, due to selection in information processing, my baseline model can generate the leptokurtic distribution endogenously even if the distribution of underlying shocks is Gaussian.

Value of Information about the Aggregate Shock Given both idiosyncratic shocks and aggregate shocks, which shocks do firms pay more attention to? Optimal attention allocation implies that firms have an incentive to allocate their attention toward more volatile shocks. This is also true in my baseline models. The first row of Table 5 shows, in both single- and two-product models, that the average Kalman gains across all firms for the idiosyncratic shocks are larger than those for the aggregate shock because the idiosyncratic shocks are more volatile than the aggregate shock.

However, the amount of information processing about the aggregate shock is different in the single- and the two-product models. The value of information about the aggregate shock is higher for the two-product firms than the single-product firms, as the firms’ frictionless optimal prices for all goods are affected by the aggregate shock. Table 5 shows that the Kalman gains for aggregates shocks are about twice as large in the two-product model than in the single-product model. This

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48 In Appendix D, I compare the distributions of actual price changes in the baseline single- and two-product models and the standard menu-cost-only models.

49 This endogenous leptokurtic distribution of perceived price gaps is also present in the two-product model as shown in Appendix Figure G.5.
implies that firms in the two-product economy will be more responsive to the monetary policy shock than firms in the single-product economy as they are more informed about it.

4.5 Real Effects of Monetary Policy Shocks

In this subsection, I show that monetary non-neutrality in the one-good version of the model is nearly as large as that in the Calvo sticky price model, while it decreases in the two-good version of the model. To show this, I take the calibrated models and hit them with a one-standard-deviation shock to monetary policy. Figure 6 shows the impulse responses of output in the one-good and the two-good versions of the model. I also show the impulse responses in the standard menu cost model with single-product firms and in the Calvo sticky price model.

The output response to a one-standard-deviation monetary policy shock in the standard menu cost model is small and short-lived. The half-life of output response is only two months. This is a well-known fact in this model: there are large selection effects of price changes, which act as a strong force to reduce monetary non-neutrality (e.g., Golosov and Lucas, 2007). In my one-good version of the baseline model with both menu costs and rational inattention however, the real effects of monetary policy shocks are large and persistent. The impact response increases by 60%.

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50In the appendix, I show how the real effects of monetary shocks in my baseline models change when I shut down one friction at a time. Appendix Figures G.6 and G.7 show the output responses in the menu-cost-only models and in the rational-inattention-only models with single- and two-product firms, respectively.
Figure 7: Counterfactuals and Model Mechanism

Notes: This figure shows counterfactual models and the implied model mechanisms. Model (1A), (1B), (1C), and (1D) are single-product menu cost models with different assumptions about firms’ information set. In model (1A), firms have full-information about both idiosyncratic and monetary shocks. Firms in model (1B) have perfect information about the monetary shock, but choose their optimal signal about the idiosyncratic shock. All firms in model (1C) are given the same exogenous signal about the monetary shock, while they choose their optimal signal about the idiosyncratic shock. Model (1D) is the baseline single-product model where all firms choose their optimal signals about both shocks. Model (2D) is the baseline two-product model. See Section 4.6 for details.

and the cumulative output responses, which are defined as the area under the impulse responses of output, are about seven times larger than those in the standard menu cost model. In fact, this large real effect is comparable to that in the Calvo sticky price model.

However, the large real effects are reduced in the two-product model. The cumulative output responses in the two-product model are 12% smaller than those in the single-product model, and the half-life of output responses also decreases from eight months in the single-product model to seven months in the two-product model. Interestingly, as shown in Appendix Figure G.3, the implied kurtosis of the distribution of price changes is higher in the two-good version of the model than in the one-good version of the model. This suggests that in this model with both rational inattention and menu costs, the ratio of kurtosis to the frequency of price changes might not be a sufficient statistic for the output response to a monetary shock, which is derived by Alvarez et al. (2016).51

51 The rational inattention models are outside the class of models studied by Alvarez et al. (2016) because of imperfect information about monetary policy shocks. In their model, the real effects are calculated by the output responses to a once and for all unexpected monetary shock that is perfectly observed by firms. In addition, after the monetary shock, firms use the same decision rule used in the steady state. In the rational inattention model, however, firms do not have perfect information about the monetary policy shocks, and their optimal policy rule depends on their uncertainty about the shocks. More importantly, firms’ optimal information acquisition decisions are affected by their product scope. This relationship makes two-product firms more informed about the monetary shock than single-product firms. Thus, monetary non-neutrality in the two-product model is smaller than that in the single-product model even if kurtosis of the distribution of price changes is higher in the two-product model. This imperfect information about the monetary shocks breaks the application of the sufficient statistics derived by Alvarez et al. (2016).
In this subsection, I investigate the key mechanisms behind the results of monetary non-neutrality in the baseline model. To this end, I start from the standard menu cost model with single-product firms, such as Golosov and Lucas (2007), and consider counterfactual models by adding core elements of the baseline model. I discuss five main mechanisms; three of them have been studied in the previous literature while two of them are new in this paper. Figure 7 shows how each counterfactual model is related to the underlying mechanisms that I discuss here in detail.

**Endogenous Leptokurtic Distribution** The first model (1A) is the standard menu cost model with single-product firms such as Golosov and Lucas (2007). In this model, firms have perfect information about both idiosyncratic and monetary shocks. As I discussed earlier, this model implies small and short-lived real effects of monetary shocks due to large selection effects of price changes (see the black solid line in Figure 8). For comparison with other counterfactual models, I normalize the impact response of output and the cumulative output responses in this model to 1.

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**Figure 8: Impulse Responses of Output in Counterfactual Models**

*Notes:* This figure plots output responses to a one standard deviation monetary shock in counterfactual models described in Figure 7. Cumulative IRFs refers to area under the output responses. I normalize both impact and cumulative output response in the menu-cost-only model with single-product firms as 1. See Section 4.6 for details.
In the next model (1B), I assume that the single-product firms have perfect information about the monetary shock but are rationally inattentive to their good-specific shock. Because firms choose their optimal signals about the good-specific shock, selection in information processing about the idiosyncratic shock makes the distribution of firms’ desired prices endogenously leptokurtic. This leads to small price selection effects because there is only a small fraction of firms around the inaction bands. Thus, the output responses in this counterfactual model are larger than those in the menu-cost-only model. As shown in Figure 8, the impact output effect in model (1B) increases by 23% compared with that in model (1A). This mechanism is new in the literature; the interaction between menu costs and rational inattention generates the endogenous leptokurtic distribution of desired price changes, which amplifies the real effects of monetary shocks in a non-trivial way.

**Imperfect Information about Monetary Policy Shocks** Next, I assume that single-product firms are not only rationally inattentive to the good-specific shock and choose their optimal signals about it, but also informationally constrained about monetary shocks. I assume that all single-product firms are exogenously given a signal about the monetary shocks. The signal has the same precision as the average precision of signals in the one-good version of the baseline model (1D). In other words, firms in this counterfactual economy have the same degree of attentiveness to the monetary shocks as do firms in the baseline model where all information choices are endogenous. This counterfactual model (1C) captures the role of imperfect information about monetary shocks for monetary non-neutrality. This mechanism has been widely studied in the literature (e.g., Lucas, 1972; Woodford, 2003; Maćkowiak and Wiederholt, 2009). Figure 8 shows that this channel has the most important role for amplifying the real effects of monetary policy shocks. The cumulative output effects are seven times bigger in this model than those in the standard menu cost model.

**Selection in Information Processing about Monetary Policy Shocks** Now, I assume that the single-product firms choose their optimal signal about the monetary policy shocks rather than receive an exogenous signal. This model (1D) is the baseline single-product model that I studied in the previous section. The comparison of this model to model (1C) captures the role of selection in information processing about the monetary shocks. Notice that firms in both models (1C) and (1D), on average, acquire and process the same amount of information about the monetary shocks. While all firms have the same amount of information about the monetary shocks in the model (1C), price adjusters choose to have better information about the monetary shocks than non-adjusters in the single-product model (1D). This implies that firms that change their prices following the

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52 As firms have perfect information about monetary policy shocks and the aggregate demand follows a random walk process, they immediately observe and respond to the shocks if their prices are around the adjustment margins, which also makes the real effects of monetary shocks in this counterfactual model short-lived.
monetary shocks in the baseline model will adjust more strongly and learn quickly about the shocks compared with the price-adjusting firms in model (1C). Thus, the real effects of monetary shocks are smaller in this baseline model (1D) than those in the model (1C). Figure 8 shows that the cumulative output responses in the baseline one-good version of the model are 20% smaller than those in model (1C). This mechanism is also new in the literature.

**Economies of Scope in Price Setting and Information Processing** Lastly, I consider the baseline two-product model. The two-good version of the model entails economies of scope motives in both price setting and information processing. Notice that both economies of scope motives work in the opposite directions for monetary non-neutrality. Figure 8 shows that the cumulative output effects of monetary shocks decrease by 12% in the two-product model compared with the single-product model. This implies that in the calibrated model, economies of scope in information processing act as a strong force to reduce monetary non-neutrality.

### 4.7 Models with a Large Number of Products

In Section 4.5, I show that the real effects of monetary shocks decrease in the two-product model compared with the single-product model. In this section, I show that this implication of multi-product pricing for monetary non-neutrality can be extended to the models with an arbitrary large number of products. The main computational challenge for solving the baseline model with more than two products is that the number of state variables double with an additional good produced by the firms and firms’ optimal information choice problem is subject to occasionally binding constraints. To simplify the analysis, I make two assumptions. First, I assume that firms choose how much to process information about the underlying shocks as if they do not face menu costs. Second, given menu costs, firms choose their prices based on that information, but they are myopic in the sense that they do not care about the continuation value of their current pricing decisions. One limitation of making these assumptions is that it eliminates the interesting interaction between rational inattention and menu costs because all firms choose to have the same information set about the underlying shocks. However, as it simplifies the model analysis by eliminating state variables but keeps the core of the baseline model, I analyze the implications of multi-product pricing for monetary non-neutrality under these assumptions.\(^{53}\)

\(^{53}\)For example, as shown in Appendix Figure G.8, the backcast errors in the growth rate of nominal GDP decrease with the number of products. This relationship stems from the economies of scope in information processing in rational inattention models with multi-product firms. Moreover, kurtosis of the distribution of price changes increases with the number of products and converges to a value of three, which is consistent with the implications of menu cost models with multi-product firms.

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Figure 9: Cumulative Output Responses and Number of Products in the Simplified Models

Notes: This figure plots cumulative output responses in the simplified models with different number of products. "RI+Menu Cost" refers to the model with both menu costs and rational inattention. Red line shows the cumulative output responses in the only menu cost models with different number of products and blue line shows those in the models with both menu costs and rational inattention with different number of products. See Section 4.7 for details.

Figure 9 shows the cumulative output responses to a monetary shock in the simplified models with various numbers of products. I calibrate each model with a different number of products to have the same size and frequency of price changes. I normalize the cumulative output response in the one-good version of the menu-cost-only model to one. In the menu-cost-only models (red line with circles), the cumulative output effects increase with the firms’ number of products. In the models with both rational inattention and menu costs (blue line with diamonds), the real effects decrease with the number of products but converge to the menu-cost-only models with a large number of products. As the number of products increases in the model, firms’ subjective uncertainty about monetary policy shocks decreases and converges to zero, implying firms have almost perfect information about the monetary policy shocks.\textsuperscript{54} Again, this implies that the ratio of kurtosis to the frequency of price changes might not be a sufficient statistic for monetary non-neutrality in the models with both rational inattention and menu costs. My main conclusion about the relationship between monetary non-neutrality and firms’ product scopes can be extended to the model with a large number of products.

\textsuperscript{54}In Appendix B.1, I show this negative relationship between the number of products and firms’ subjective uncertainty about monetary policy shocks.
5 Conclusion

Understanding the nature of firms’ expectations formation and price setting behavior has been an active area of research in monetary economics. The first part of this paper uses a firm-level survey from New Zealand to show how firms’ product scopes are related to their expectations formation and price setting behavior. I find two stylized facts: firms with a greater number of products have both 1) better information about aggregate inflation and 2) more frequent but smaller price changes.

The second part of the paper then builds a dynamic general equilibrium menu cost model with rationally inattentive multi-product firms to study the aggregate implications for monetary non-neutrality. In this model, the interaction between nominal and informational rigidities gives rise to a novel selection effect of information processing: price adjusters have better information about the underlying shocks than non-adjusters because firms with higher subjective uncertainty are less likely to change prices and want to wait until they acquire more information and resolve their uncertainty. I show that this new selection effect leads to a fat-tail distribution of firms’ desired price changes and thus weakens selection effects of price changes. As a result, the real effects of monetary policy shocks in the one-good version of the model are nearly as large as those in the Calvo model. Finally, I show that in the two-good version of the model, the cumulative output effects decrease by 12% compared with the one-good version of the model due to the strong economies of scope motive in information processing.

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